Supplementary Information for
Multi-parameter encrypted orbital angular momentum multiplexed holography based on multi-ramp helico-conical beams

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Supplementary Text

Note S1: The influence of parameter r_0 on sampling constant d

The sampling constant d is determined by the spatial frequency distribution of the phase distribution of the MHC beam. However, in addition to α and m, different normalized factor r_0 also affect the MHC phase distributions resulting in different spatial frequency distributions, as shown in Figs. S1(a) and (b). The sampling constant d as a function of normalized factor r_0 is shown in Fig. S1(c). It can be seen that there is a nonlinear relationship between r_0 and d, and d will decrease as the r_0 increases. The fitted curve is: $d=41.436+894.708\exp(-r_0/0.175)+293.148\exp(-r_0/0.751)$.

Note S2: The influence of parameter m on MHC-OAM multiplexed holography

Here the crosstalk in the MHC-OAM multiplexed holography is numerically investigated to analyze the influence of parameter m on the results. The signal-to-noise ratio (SNR) is used to evaluate the crosstalk, as shown in Fig. S2. An MHC-OAM multiplexed hologram is obtained by encoding four images and four different MHC phase modes with different m. When the m-encrypted MHC-OAM multiplexed hologram is illuminated by a correctly incident MHC beam with a customized m, the desired target image with a relatively strong peak intensity can be obtained and exhibits the strong mode selectivity. The simulation results show that using MHC modes with a difference $\Delta m=1$ could produce low crosstalk in holographic multiplexing.

Note S3: The influence of parameter r_0 on MHC-OAM multiplexed holography

The MHC beams with differences of $\Delta r_0=0.05$ mm, 0.1 mm, and 0.15 mm are used to encode and multiplex four images. The reconstructed SNR is shown in Table S1, and the reconstructed target images are shown in Fig. S3. Our numerically results show that a lager r_0 difference can obtain a lower crosstalk. Note that the decrease in SNR for MHC modes with a smaller r_0 difference is due to the spatial overlap of these modes. In addition, the target images with larger sampling constants are numerically simulated as shown in Table S2 and Fig. S4. It can be clearly seen that each reconstructed image has a relatively good SNR, which is due to the reduced spatial overlap of the modes. Our numerical results suggest that the selection of larger sampling constants is beneficial to improve the SNR and to ensure that the target image can be recognized.

Table S1 Numerical comparison of the parameter r_0 on the reconstructed results.

Δr_0	SNR(dB)			
0.05 mm	4.17	2.08	3.09	4.93
0.1 mm	5.27	4.71	5.53	6.51
0.15 mm	5.98	5.63	6.97	8.83

Table S2 Numerical comparison of the parameter r_0 on the reconstructed results with larger sampling constants.

Δr_0	SNR(dB)			
0.1 mm	12.80	8.86	9.26	9.13
0.15 mm	13.57	10.07	9.48	9.38

Note S4: Crosstalk analysis in MHC-OAM multiplexed holography

The MHC-OAM multiplexing holograms used could be represented as the superposition of the electric fields of different images that are encrypted into different images channels. Therefore, the phase-only multiplexing hologram can be described as:

$$P = \arg\left[\sum_{i=1}^{N}\exp(i\psi_i)\exp(i\phi_i)\right] \tag{S1}$$

and the electric field can be expressed as:

$$E = \sum_{i=1}^{N}\exp(i\phi_i)\exp(i\psi_i) \tag{S2}$$

where ψ_i and ϕ_i represent the phase information of each image channel and the encoded MHC phase distribution, respectively, and N indicates the number of multiplexing channels. ψ_i can be obtained by the adaptive weighted Gerchberg-Saxton (AWGS) algorithm. Here, the crosstalk among OAM image channels is still a problem. This is due to the presence of undesired image channels with low peak intensities in the background. As an example, the desired image channel is encoded by an MHC mode with phase distribution of $\phi(\alpha, m, K = 0, r_0)$. The phase and intensity distributions of each pixel in the reconstructed images using different MHC modes with phase distributions of $\phi(-\alpha, m, K = 0, r_0)$ and $\phi(-\alpha, m, K = 1, r_0)$ is shown in Fig. S5. It can be seen that when the hologram of desired image channel is illuminated by a correctly incident MHC beam with $\phi(-\alpha, m, K = 0, r_0)$, the Gaussian mode with a stronger intensity distribution in each pixel of desired image can be obtained, while an interference mode with lower peak intensity will appear, which can be regarded as background disturbance, resulting in the generation of the crosstalk.

Note S5: The influence of parameter α on MHC-OAM multiplexed holography

As an example, the MHC beams with a topological charge difference of $\Delta\alpha$=1, 2, 3 are used to encode and multiplex four images, and the results are shown in Table S3 and Fig. S6. Our numerical results show that the use of MHC beams with a larger TC difference in the holographic multiplexing could yields a lower crosstalk. Note that the target image “L” has a large crosstalk because of the relatively large spatial overlap of the MHC modes. To improve the SNR, the Gaussian mode selective aperture array can be used.[15]

Table S3 Numerical comparison of the parameter α on the reconstructed results.

Δα	SNR(dB)			
1	5.16	6.83	7.21	8.11
2	5.42	7.34	9.52	10.17
3	5.75	8.56	10.16	12.64

Note S6: The influence of parameter *K* on MHC-OAM multiplexed holography

Here, the MHC modes with different constant *K* are used to reconstruct the information in holographic multiplexing. As an example, two MHC phase modes $\phi(\alpha, m, K = 0, r_0)$ and $\phi(\alpha, m, K = 1, r_0)$ are used to multiplex two encoded images ‘Z’ and ‘N’. Two *K*-encrypted MHC-OAM holograms can be obtained, which are superimposed into a *K*-encrypted MHC-OAM multiplexing hologram. When the hologram is illuminated by different MHC beams with different value of *K*, namely, $\phi_{de}(-\alpha, m, K = 0, r_0)$ and $\phi_{de}(-\alpha, m, K = 1, r_0)$, two target images can be obtained, respectively. The simulation results are shown in Fig. S7.

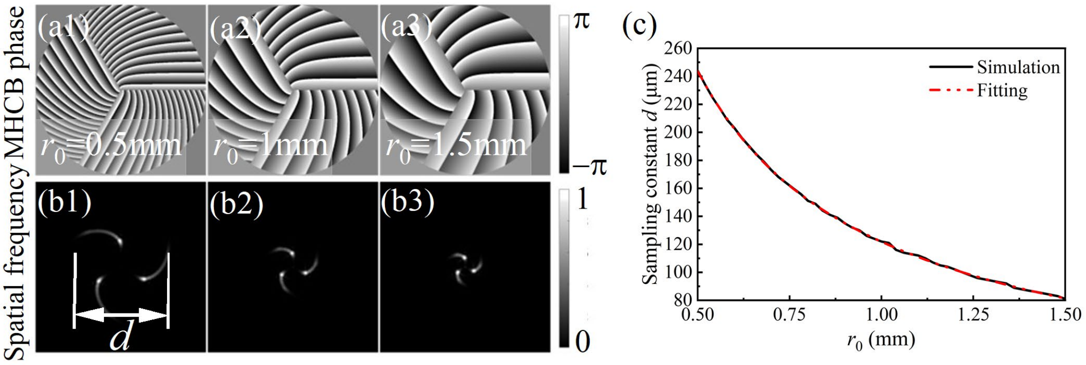


Fig. S1 (a1)-(a3) The phase distribution of the MHC beam with $\alpha = 5$, $m = 3$. (b1)-(b3) The simulation of the spatial frequency distribution of the MHC beam. (c) The relationship between the r_0 and sampling constant d.

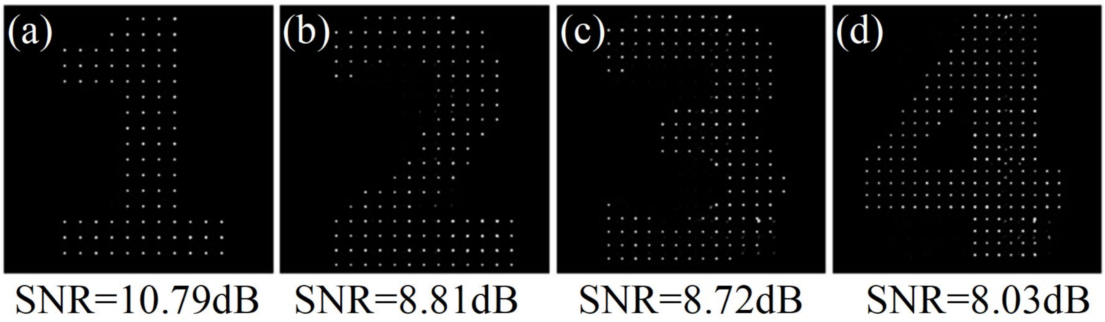


Fig. S2 Numerical reconstruction results based on different encoding parameters (a) m=4, (b) m=5, (c) m=6, and (d) m=7.

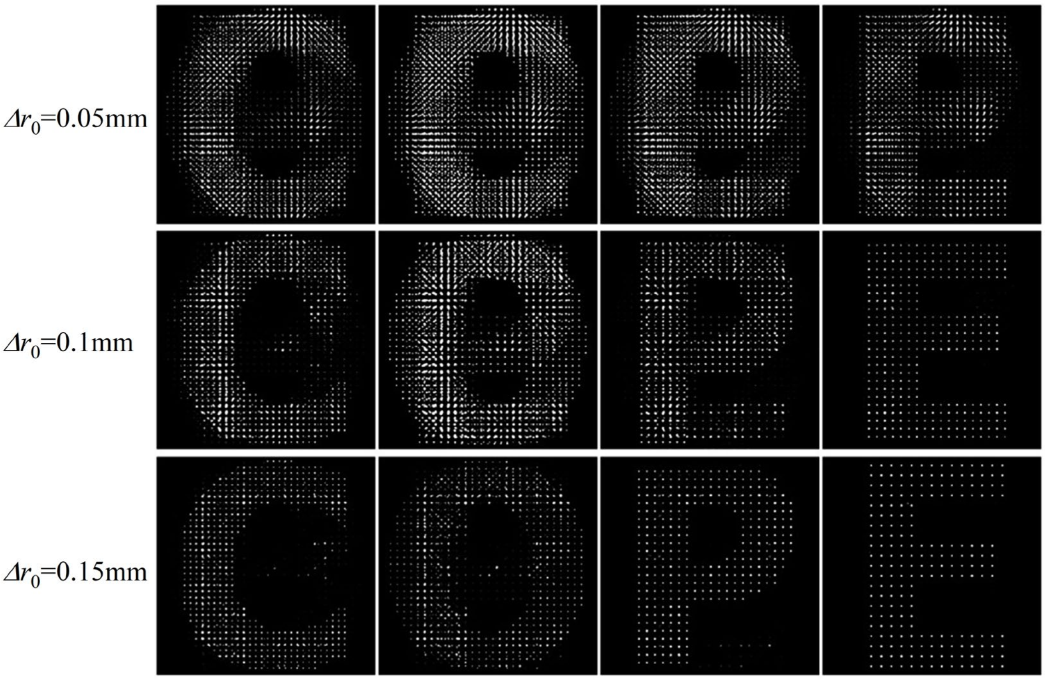


Fig. S3 Numerical reconstruction results based on different encoding r_0 differences. The r_0 differences of the first, second and third rows are 0.05 mm, 0.1 mm and 0.15 mm, respectively.

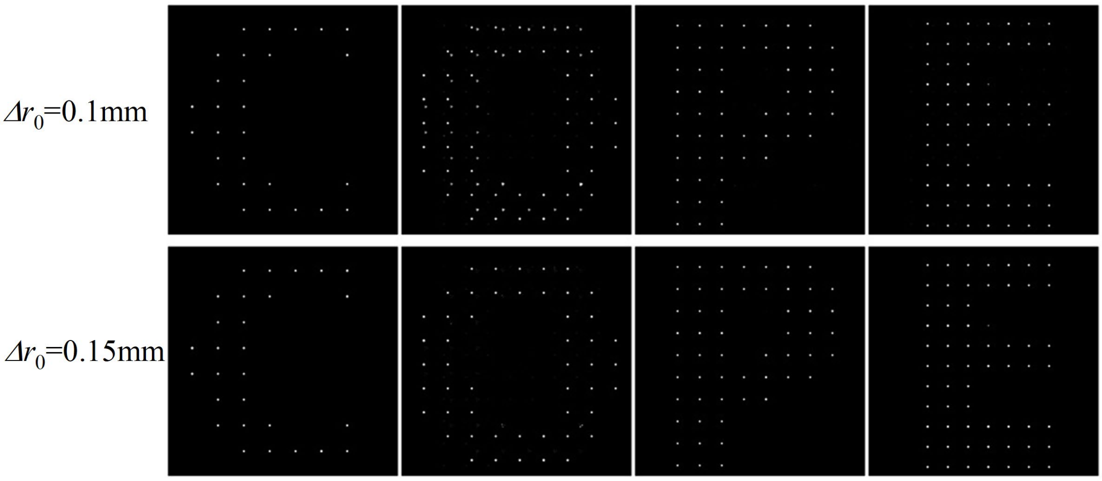


Fig. S4 Numerical reconstruction results with larger sampling constants. The r_0 differences of the first and second rows are 0.1 mm and 0.15 mm, respectively.

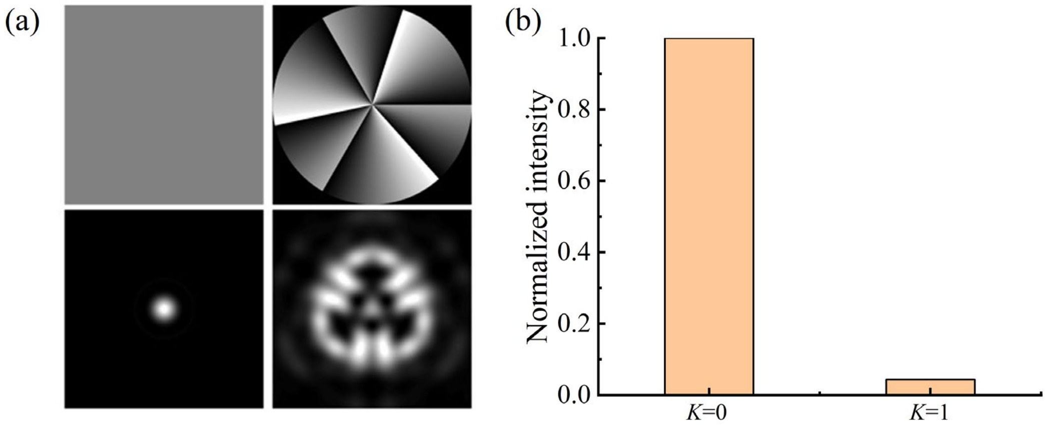


Fig. S5 (a) Numerical characterization of the MHC mode conversion. The phase and intensity distributions of different incident MHC beams in each pixel of the reconstructed image are given. (b) The normalized intensity $I/I_{Gaussian}$.

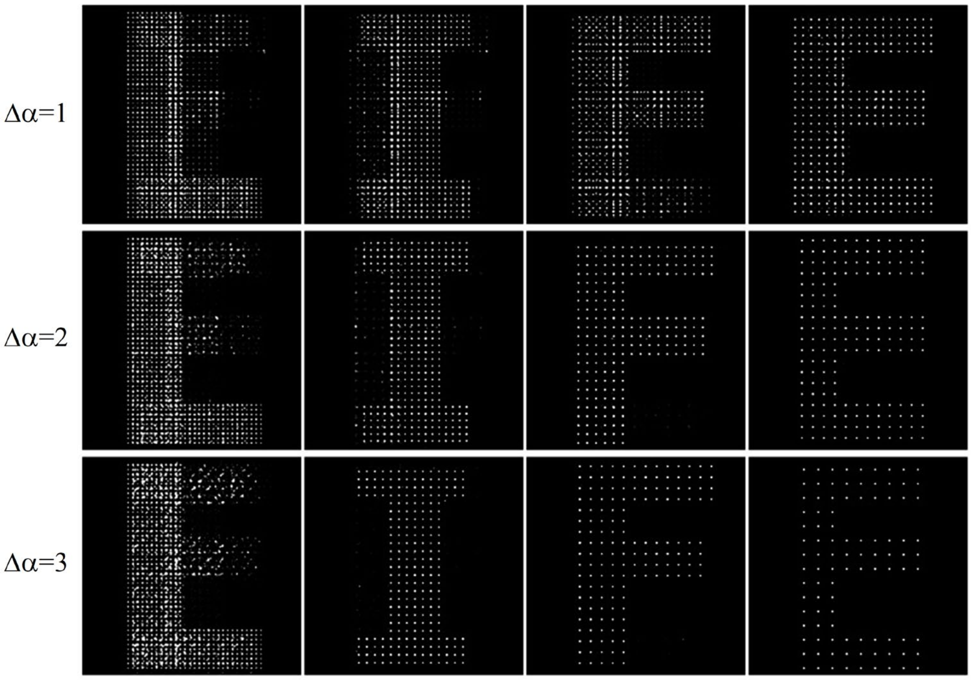
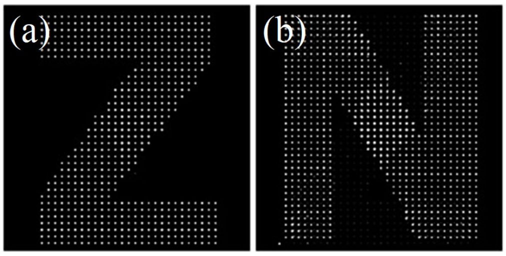


Fig. S6 Numerical reconstruction results based on different encoding TC differences.



SNR=10.23dB SNR=8.56dB

Fig. S7 Numerical reconstruction results based on different encoding parameters (a) *K*=0, (b) *K*=1.